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EDITORIAL

THE PRIMARY SCHOOL

Education is a long and complex process which extends from birth to death. Somewhere along that line one receives what might be termed formal education in school, college or university; but this is only a specialized part of a life-long process.

Mathematics is so much a part of life that one's mathematical education may likewise be said to extend throughout life. Where do one's ideas on number really originate, and when do these ideas take root? Psychologists may essay their guesses, but can they really say when the child is first aware that he has *two* hands, that he has *two* parents, that he is playing with *two* toys and so on? Such awareness clearly forms part of early life-experience. But awareness of a thing is not the same as understanding it, and the development of these early ideas, their generalization and their application to operational processes takes place once formal education starts in the Primary School. Here, in the Primary School, is laid the true foundation of all mathematical learning.

What, therefore, should we expect the Primary School to give in the way of mathematical education? Ask a Secondary School teacher what he feels a pupil should know when he arrives at the Secondary School stage, and you would be told that the real essential was a knowledge of the first four rules; other topics (e.g. fractions, decimals) might be added according to the ideas of the particular individual, but this would be the type of answer that one would receive. However, these are actual topics such as one would find in a textbook; are they enough? Is this all we need expect of the Primary School pupil? Or is there something more to be expected than the mastery of the technique of doing 'sums'?

Mathematical processes, if they are to be taught at all, should be understood. The child in the Primary School should appreciate the true meaning of simple mathematical operations and obtain a real understanding of number, quantity and space. These are the fundamentals of mathematics, the foundations on which all future work is built, and upon successful teaching of this basic work—rather than the mastery of the four rules as such—depends future success in the subject. But even more than this should be expected of Primary School mathematics. The child should begin to see the need for the subject and realize its importance in life and work; he should begin to feel an appreciation of, and liking for mathematics; and coupled with facility in the subject should come the satisfaction of achievement.

For in the Primary School is laid not only the foundation of the subject, but also the foundation of the student's attitude to the subject. In fact, a dislike or a distrust of the subject may be inculcated here which does not become evident until much later in the child's educational career. This is not, of course, to excuse bad and ineffective teaching at other levels, but rather to emphasize that this is the formative period of education. Not without reason did Pestalozzi speak of his kindergarten—'garden of children'—with all that it implies.

Consequently, the problem of what and how to teach in the Primary School is one which is the vital concern of all teachers at every level of the educational system. In this and future issues of *Mathematics Teaching* it is our intention to devote increasing space to the discussion of the work of the Primary School. This does not

mean less space available for other features, but if it did it would not imply less material to concern those who did not teach in this type of school; it is essential that others read and reflect on these articles and interest themselves in what is going on in the Primary Schools, contributing their own ideas, where possible, on the subject.

In the same way, of course, it is necessary for the teacher in the Primary School to take an active interest in what goes on in Secondary Schools and in further education. What is taking place there is important to him since it is merely a continuation of a process which he has initiated; the fruitful conclusion of the work of the Primary teacher is not at 11 years of age, but many years later.

For education is a whole; we split it up for ease of administration and in the cause of specialization. But let us not forget that a child receives only *one* education, and it is for those from whom he learns to co-operate in ensuring a unity, continuity and effectiveness in bringing about his fullest mental and physical development.

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PRACTICAL MATHEMATICS IN THE PRIMARY SCHOOL.

A. L. OGLE.

"Comparison of the arithmetic quotients with the intelligence quotients of pupils on entry to the modern schools indicated that the pupils with higher intelligence quotients had not reached the full extent of their potential in mathematics in the primary school," states the report *The Teaching of Mathematics in Manchester Secondary Modern Schools*[1], which was reviewed in the July issue of *Mathematics Teaching*. The report also says that modern schools are handicapped by the shortage of teachers qualified to teach mathematics. One out of every two teachers was expected to teach mathematics; three out of every four who taught the subject had no special qualifications in it. Several points arise worthy of consideration in relationship to primary school mathematics.

Firstly, "mathematics" should be taught in the primary school and is indeed being taught, whatever it is called on the timetable—one West Country headmaster told me recently that he would not deem to call the arithmetic in his school "mathematics," but surely if his children are able to appreciate the fact that $9 - 6=3$ is opposite to, and is implied in $3+6=9$, they are thinking mathematically.

Secondly, if primary schools are sending children to modern schools not stretched to the full extent of their potential in mathematics what of the primary school children admitted to grammar schools? Are these children to be handicapped by the limit to which their primary teachers can take them? If one out of every two teachers in the secondary modern school is expected to teach mathematics every teacher in the primary school is compelled to do so. If three out of four who teach the subject in the secondary school have no special qualification what proportion in primary schools have neither liking nor qualification?

Thirdly, we must ask ourselves "how important is mathematics in the primary school in relationship to other subjects in the curriculum?" We should like to be experts in all the subjects which we teach, history, geography, science, English literature and grammar, physical education and religious instruction. As we have not the time, even if we have the inclination, we must choose which we consider is the most important both to our pupils and ourselves. Obviously those primary school teachers reading this are concerned about the dilemma.

There is need for immediate action both in our own self-education and in starting to change our classroom habits. Pages and pages of sums are done religiously every day by millions of primary school children who have learnt to do arithmetic by drill methods without possessing an understanding of number. Too many of us have relied upon the "little sum cards" mentioned by E. M. Churchill when she criticizes Piaget for not paying sufficient attention to the influence of cultural and educational factors on the developmental process. She feels that if children have as little understanding of the meaning of numbers as Piaget suggests then the children are wasting their time with these sums and could be more usefully occupied in other activities.

The blame for this state of affairs is often levelled at the Junior School. On the whole infant techniques aim to give the child a practical course of number work. An interesting point raised by J. B. Biggs in his article *The Development of Number*

Concepts in Young Children[2] is that there is some evidence that an insistence on "enlivening" number work which is apparent in many infants' schools may actually impede the young child's progress in understanding the essential constancy of number meanings. The principle of conservation, as this development is called, is one well worthy of discussion, perhaps in this journal at a future date. Nevertheless, whether infants have been trained on buttons, milk bottle tops or on the Cuisenaire or Stern material, it is generally admitted that they come to the Junior School with a body of practical experience only to be plunged immediately into clouds of abstraction.

The justification for so much work of a formal nature is often given as the need to get the child ready for the Selection Examination at the end of their four years in the Junior School. There could well be a profitable study of the arithmetic tests set by the authorities. Enlightened Authorities, however, are relying less and less on mechanical tests of arithmetic which are, by themselves, inadequate measures of ability to "do" arithmetic. If you are unhappily situated in an "unenlightened" authority, perhaps by your efforts and those of your fellow sympathisers, together with the A.T.A.M. something can be done about it. But, very often, the sole excuse for poor teaching of practical mathematics in the junior school is our own inability to do something about it.

I give below some ideas for a practical approach to mathematics in the junior school. It is not suggested that these will be quick and easy means of "stretching the potential in mathematics." They will be useful if they do nothing but cause the reader to think about his own presentation of the subject and whether it is "learner-centred." Perhaps those who have tried all these or know of others will share their experiences with us.

Like the medieval chroniclers, I have "heaped together" all that I could find and no attempt has been made to group these suggestions at ability levels; this the teacher must tackle empirically.

Weather.

1. Daily readings of the classroom temperature recorded on to graphs. The seven year old making block graphs; the eleven year old calculating average temperatures over the week or month and working out comparisons from one year to another if the recording is kept in a book.
2. An outside weather station maintained. Maximum and minimum temperatures recorded and averages taken. Rainfall noted and the average for the month and year calculated. Wet and dry bulbs set up in a Stevenson screen and the percentage of humidity worked out.
3. Barometric pressure recorded and compared.
4. A gnomon could be set up either indoors or out. Some interesting patterns may be made from the lines drawn along the shadows cast at hourly intervals. This has all sorts of possibilities.
5. Calculating heights from shadows[3].

About themselves.

1. Each child to have a card on which his weight and height are written. This to be maintained every six months throughout his junior school career.

2. A wall chart of the height of each child in the class and comparison of growth recorded[4].

Classroom shops.

Much has already been written about these. Interest will be sustained by the teacher and the children if stock is turned over regularly and different packages put into the shop. Bills involving many items should be written out and change given. Is it asking too much for real money to be used in this activity?

Weights and Measures of capacity.

Need these teaching aids be confined to the lower forms only? Children work out complicated mechanical sums involving hundredweights and quarters but have they ever seen a hundredweight?

The education section of the British Association discussed "science and number in the primary school" during their meetings held this year and Mr. N. F. Newbury, the director of education for St. Helens, suggested that each child should be faced by genuine problems in science in keeping with his ability and maturity[5]. Many scientific interests in the junior school are closely linked with mathematics: would children be interested in the tensile qualities of various threads tested with a spring balance? What is the Plimsoll line and how does it work?

Household Bills.

Make a collection of gas and electricity bills from parents. The children check the therms and units consumed with the amount charged.

Problems involving time, distance and speed.

Think about helping children to understand these quite difficult concepts by getting them out on to the playground or field and timing the speeds of walking, running and bicycling over set distances.

Timetables.

Ask a bus company for a set of timetables; those printed on separate sheets are best. Much work involving time and mileage, perhaps of a formal nature can be done on these. If timetables from various airline companies and shipping lines are collected, separate assignment work can be evolved by the teacher.

Let's Be Practical[6], a system of work cards designed for secondary modern pupils but suitable for many in the junior school, makes a good starting point. The ideas in this series of work cards are capable of further development.

The Story of a Family.

Plenty of arithmetical situations arise around a family! Create an imaginary family and a series of work cards involving the happenings in their lives, e.g.

1. About themselves; heights, ages, weights. Use graphs to show development.
2. Their income; salary, pocket money, income tax, savings, expenditure, national insurance, unemployment pay, sickness benefit, hire purchase repayments.
3. Their house; mortgage, insurance, rates, gas and electricity, water rates, furniture, decoration.

4. Budgets; clothes, food, entertainment, holidays. Get as many catalogues as possible and invent situations from these. Holidays abroad involving changes from one currency to another are situations well within the experience of the junior child these days.
5. In the kitchen; which is the best refrigerator, gas cooker, electric fire, etc., to buy? The Consumer Advisory Council publish books about these matters and the children can assess relative prices and values.
6. In the garden; some useful work here on scale drawing and area.

The possibilities of this scheme seem endless; I saw it used in a small school in Sussex but as the headteacher remarked, "the main disadvantage is the time and care necessary to ensure reasonable progression and suitability of examples."

By exploring some of the ideas given here, teacher and child together should become aware of the mathematics all around them and find pleasure in the discovery.

In conclusion I quote Mr. Newbury who said that the child should not be told the answer, but should solve the problem by personal experience involving doing, observing, investigating, recording, measuring, and thinking. And Dr. Gattegno, our President, who says, "look around and you will find much to do."

References.

1. **The Teaching of Mathematics in Manchester Secondary Modern Schools** —published by Manchester L.E.A.
2. Article by J. B. Biggs **"The Development of Number Concepts in Young Children"** in the February issue of Educational Research—published by Newnes.
3. Calculating shadows—page 129, **Juniors Learning Mathematics,** Barnet, Faithfull and Threakston—published by Educational Supply Association.
4. A new series of charts by John Lunn is being published.
5. Review of the British Association Meeting in the Times Educational Supplement of 11th September, 1959.
6. **Let's be Practical,** by Alan J. Gould—published by Chambers.

OUR NEXT ISSUE.

The next issue of *Mathematics Teaching* will appear in March and will feature a consideration of Sixth Form work, including the report of this summer's International Conference on mathematics teaching which considered the transition from School to University. There will be articles on aspects of work in Primary Schools, and on a new approach to the work of the first year in the Secondary Modern School. Among other features are articles on model-making, book reviews, and a new section reviewing apparatus. The March issue will be sent free to all members of the Association. *Make sure of your issue by sending your subscription for* 1960 *to the Treasurer at once;* the cost is only ten shillings.

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TEACHING AIDS AND LOGIC
I.—PUZZLES IN THE CLASSROOM

TREVOR FLETCHER

'Mathematics provides a training in logic'—one often hears teachers justify their subject by making this claim. But how true is it that our present teaching of the subject actually does so? University teachers often complain that pupils coming up from schools have little understanding of logical rigour, and fail even to see the need for it; and they are speaking of the most successful pupils from the Grammar schools, they are not referring to average pupils, still less to the below average.

Two generations ago 'training in logic' often meant the memorising of certain geometrical theorems, whether they had been understood or not, and reproducing them letter for letter. The folly of this type of training has long been recognised, but its spectre remains because the idea is still widespread that logic in mathematics really is best exemplified by theorems of this kind (if only pupils are good enough to understand them) and the 'proper' way to teach logic in the classroom is to tend as closely to old-fashioned geometry as one can. But just as geometry is no longer what it was in Euclid's time, so logic is no longer what it was in Aristotle's.

Two aspects of logic are important in teaching. The teacher should know something of contemporary views of logic as a purely deductive science, and of its relations to philosophy and mathematics [1], [2], [3]; and also he should continually consider the subjective aspects of reasoning—how individuals learn to reason logically and how they apply this power in their daily problems.

Polya says [4], 'It seems to me that mathematics is, in several respects, the most appropriate experimental material for the study of inductive reasoning. This study involves psychological experiments of a sort: you have to experience how your confidence in a conjecture is swayed by various kinds of evidence. Thanks to their inherent simplicity and clarity, mathematical subjects lend themselves to this sort of psychological experiment much better than subjects in any other field.' The powerful and provocative ideas which Polya puts forward in his books [4], [5], should be studied by all teachers; but for the moment our main concern will be with the cultivation of an appreciation of deductive logic. We will consider how elementary puzzles and pieces of apparatus can provide situations in which logical relations, as it were, crystallize out and become part of the pupil's store of experience, on which later a fuller understanding of abstract systems can be based.

We hope that this will be the first of a series of articles by different authors devoted to teaching aids in logic. In this one we discuss the part which puzzles can play in cultivating an appreciation of structure, and later articles will deal with the logical relations exemplified by electrical circuits, and logical computation by punched cards and similar devices.

Of course, teachers must continually seek to improve their own theoretical knowledge of logic. We have said that logic is not what it was in Aristotle's time; neither is it what it was a few decades ago, and unless the teacher has some appreciation of modern trends [6], [7] many of the suggestions in this series will seem to be isolated tricks and gimmicks, suitable as end of term entertainment, but unworthy of any place in a systematic course of instruction. In this article we consider how puzzles may be used in class teaching with a particular logical aim in mind.

Children who are learning geometry often find it difficult to arrange the steps of a proof in the correct logical sequence. They are faced with a figure which they have to describe. They know all the time that two particular angles are equal, but they are not allowed to say so until they have first stated that two particular sides are equal. It is only natural that some should regard this as a purely arbitrary, conventional restriction upon their freedom. But the essential point of many puzzles is precisely the same—the steps have to be got in the right order, and pupils know that people do puzzles for fun, and often do so themselves. Here, then, is a selection of puzzles in which the essential point is to get the steps in the right order, and in each one pupils can readily perceive the errors resulting from trying to do things the wrong way round.

The puzzle about the wolf, the goat and the cabbage is an old favourite. A man wishes to cross a river in a boat. He has a wolf, a goat and some cabbages. The boat is only big enough to take one item of cargo each time, and if he leaves the wolf and the goat on a bank together the wolf will eat the goat, and if he leaves the goat and the cabbages together the goat will eat the cabbages. How can he get them across the river? This puzzle is typical of many river crossing situations which occur with varying degrees of difficulty.

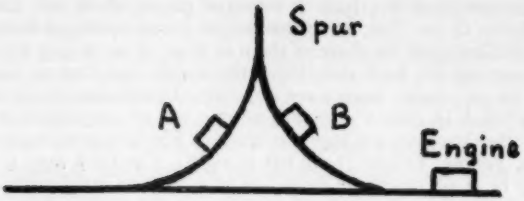
Missionaries and cannibals provide variations. A party of five missionaries is on one bank of a river, and a party of five cannibals is on the other. Each group wishes to cross to the other side, and the only means of crossing is a boat that will hold no more than three. Only one cannibal knows how to row and only one missionary, and a system of crossing is to be arranged whereby the cannibals never outnumber the missionaries either in the boat or on either bank. The solution of this takes a little longer.

Incidentally, proving that things cannot be done is often much harder than proving that they can. If there are six missionaries and six cannibals, prove this problem is impossible.[6]

Another family of problems in which the correct ordering of operations is vital is the familiar kind 'Use a five pint jug and a three pint jug to measure out four pints', or 'Use a nine pint jug and a four pint jug to measure out six pints'. As puzzles alone these are entertaining; but the situation presents a great deal of mathematics, and the better pupils can be encouraged to make up similar questions of their own. This situation is extremely rich; the graphical illustration of the problems leads naturally into the use of areal coordinates, and the discussion of what is possible in the general case could lead to Euclid's algorithm in number theory, or the concept of algebraic ideals.

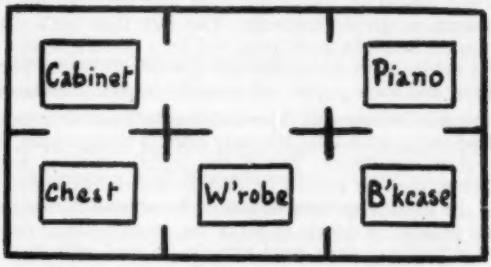
Puzzles involving rearrangement provide a range of exercises which are sometimes quite difficult. This diagram presents a situation which is basically simple, but the solution involves careful thought, including intelligent anticipation of consequences, and the direction of steps towards a particular goal (both factors of prime importance in the solution of geometrical riders).

A and B are two trucks in a siding, and the engine has to interchange their positions by legitimate shunting. The spur is only big enough to accommodate one truck at a time, and whilst one truck will go up one branch, into the spur, and down the other branch, the engine is just too big to do this. (Naturally the problem is only



solved if the engine ends up on the main line ready for further use—it is no good having it inextricably jammed in the siding.)

The next problem is by that master of puzzles, H. E. Dudeney[9].



The piano and bookcase have to be interchanged, and the rooms are so small that each will only take one piece of furniture at a time. What is the way to solve the problem in the smallest number of moves? Dudeney's many books [9], [10], [11] can be a splendid source of problems of all kinds and grades of difficulty, and many of them have a poetic originality in the mathematical concepts which they involve.

In these problems pupils can see the need to get the steps in the right order because a temporal sequence is involved. This temporal sequence is lacking in classical geometrical theorems, and this factor makes them difficult for a beginner. The strategy underlying the use of puzzles in teaching logic is gradually to replace temporal sequence by logical necessity; so let us move on to a type of puzzle presenting a new feature. The prototype concerns prisoners in a room with marks on their foreheads. Seeing the marks on the other men's foreheads they have to infer the mark on their own. These problems are more difficult, because while a temporal sequence is involved the ability to make hypothetical arguments ('if I were so-and-so then he would do do-and-so') is involved as well. The classical problem of the prisoners in a room presents a highly complex situation which I have discussed elsewhere [12], and the whole foundations of the argument on which the solution depends are open to criticism. But there is a related type of problem in which the 'prisoners' have varying degrees of information, and the logic of these is above suspicion. One example has been described in an earlier number of this journal.

The teacher prepares five hats of coloured paper, three red and two yellow. The hats are shown to the class, and three pupils are selected and seated one behind the other. We will imagine the three of them in front of us, facing from left to right. The left-hand one can see both the others, the middle one can see only the one on the right, and he, of course, cannot see anybody. Three hats are then selected and placed on their heads in such a way that no one can see the colour of his hat as it is put on. The other two hats are kept out of sight. The best combination to take first is perhaps Red, Yellow, Yellow (from left to right). The back man is then asked to say what colour he is. Of course he should be able to do so quite easily. He only *needs* to state that he knows what he is; but depending on the ability of the class it may or may not be desirable to get him to announce his actual colour. We will assume that we are working with a good class, and take it that everybody can see that the back man has decided his colour and decided it correctly. The middle man is now asked if he can decide what colour he is. After a little reflection he should be able to do so. Do not let him say his actual colour out loud, but let everyone be satisfied that it is within his powers to decide correctly. The fact that both the back ones have decided is now passed on to the front man, and he is asked if he can decide his colour. He has sufficient evidence to do so, but the position is rather harder than it is for either of the others, and some pupils will consider it quite a feat to be able to decide.

Having done this, repeat the experiment with another combination of hats—Red, Red, Yellow (say). This time the only correct thing which the back man can say is that he does not know. But if the middle man is told that the back man does not know, then he should be able to decide correctly. If these decisions are passed on to the front man then he should be able to decide as well. In fact the front man should be able to decide in all of the seven possible cases. Some teachers have found that it makes a very good exercise to deal with two or three cases in class, and then ask the pupils to work out all the others and the decisions which can be reached as homework.

A remarkable thing about this exercise is the passion which it can arouse. Pupils can become most impatient with one another, and surprisingly shamefaced when they perceive their own errors. It is clear that many problems of this kind can be made up, with a great range of difficulty.

When teaching we can pass from problems of this type to problems in which the temporal sequence is still less obtrusive, and in which there is still greater emphasis on the investigation of concurrently existing possibilities. Without attempting a detailed morphology of them we give a particularly ingenious example taken from a recent book by C. R. Wylie [8], which can be recommended as one of the best books of puzzles for mathematicians ever compiled.

I have three boxes, one of which contains two white balls; one, two black balls; and the other, one white ball and one black. There are three corresponding labels, but unfortunately the labels have all been stuck on the wrong boxes. I wish to decide which box is which by making a number of trials. Now a trial is to consist of taking one ball from a box, noting its colour, and replacing it. (If I elect to sample a box a second time there will be no means whatever of knowing whether the ball I withdraw is the one I withdrew the first time or not, if the colours are the same.) The problem is to specify a procedure by which one may decide which box is which in the minimum number of trials. It may surprise many to learn that the number of trials needed is,

in fact, only one. This situation has a certain similarity with the hat one; but, as we observed, temporal sequence is not such a strong feature.

After this one could introduce a great many puzzles with static situations which demand analysis; such as the inferential puzzles in which individuals have to be identified from statements about them, which, in some cases, may be true or false. The many puzzle books by Hubert Phillips and by Williams and Savage provide plenty of these, and we may discuss various methods of solving them in a later article.

To make systematic instructional use of puzzles one must move from problems which involve a sequence of physical actions to problems which involve the mental consideration of simultaneous hypotheses. This method rests on sound psychological foundations, because the development we advocate is precisely the development from sensory-motor action to conceptual thought which is described by Piaget.[13]

A good puzzle is to be treated with respect, because it is a microcosm of mathematics.

References:

1. Tarski: **A Modern Introduction to Logic,** O.U.P.
2. S. Langer: **Symbolic Logic,** Dover.
3. Hilbert and Ackermann; **Foundations of Theoretical Logic.**
4. G. Polya: **How to Solve it,** Princeton University Press, 1945.
5. G. Polya: **Mathematics and Plausible Reasoning,** O.U.P, 1954.
6. L. Felix: **L'Aspect Moderne des Mathématiques.**
7. Kemeny, Snell and Thompson: **Introduction to Finite Mathematics,** Prentice Hall.
8. C. R. Wylie: **101 Puzzles in Thought and Logic,** Dover, 1957.
9. H. Dudeney: **Amusements in Mathematics,** Nelson, 1917.
10. H. Dudeney: **Canterbury Puzzles,** Nelson.
11. H. Dudeney: **Puzzles and Curious Problems,** Nelson, 1932.
12. T. J. Fletcher: **The *n* Prisoners,** Mathematical Gazette, No. 332, May 1956.
13. J. Piaget: **La Psychologie de l'Intelligence,** Armand Colin, 1949.

An attempt is being made to follow up these ideas in the classroom and to obtain supplementary material and practical ideas. Anyone who is interested is asked to get in touch with I. C. N. Bell, 55 Anson Road, London, N.W.2.

BACK NUMBERS.

From time to time we receive enquiries for back numbers of *Mathematics Teaching*. At present there are still copies available of issue No. 5 (November, 1957) and all subsequent issues. The price of No. 5 is three shillings, numbers 6 to 8 cost 2s. 6d. each, and numbers 9 and 10 are 3s. 6d. each. Copies may be obtained from the Secretary.

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M. KLINE, B.SC. (MATHS. HONS.)

Senior Mathematics Master, Osmondthorpe Secondary School, Leeds

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BOOK 1, *November* 1959 BOOK 2, *Early* 1960 BOOK 3, *Spring* 1960

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The Teaching of Mathematics in the Secondary Modern School

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SECONDARY MODERN SCHOOL MATHEMATICS — II.

THE ORGANISATION OF THE MATHEMATICS COURSE.

DAVID T. MOORE.

There are two factors which Secondary Modern Schools have in common. The first is the need to provide a mathematics course for the 75 per cent of the child population who are not destined for an examination. The second is the shortage of specialist mathematics teachers. If we are to cope with the challenge to provide a stimulating course for all pupils of the wide range of ability found in Secondary Modern Schools it is essential that at least one member of the staff should be prepared to give a great deal of time and thought to the content of the course and methods to be adopted, since many teachers of the subject will be such as a matter of duty rather than from preference. I shall return to this point later.

Descartes tells us that in order to make progress we must at some time doubt everything. An attitude of honest doubt is essential when considering the question of a mathematics syllabus for Secondary Modern Schools. Nearly all of us are conditioned by tradition and our own education into feeling that the G.C.E. provides a syllabus which is fundamentally "best" or "right and proper" and that accordingly our aim should be to bring our pupils as nearly as possible to this level. This, I feel, is a fallacy. We must go right back to fundamentals; not the bases of mathematics, but the essence of the situation. Despite the growth of G.C.E. classes in Modern Schools, the vast majority of our children will never contemplate this examination. Equally they will never find any need in their adult lives to "use" much of the mathematics they learn. Why then do we try to teach mathematics at all? This question has really only arisen since 1945 when for the first time we began to try to teach mathematics to all children, instead of aiming only at proficiency in reckoning for those not taking School Certificate. Reasons for teaching the subject have been fully discussed in many places, and in detail by R. H. Collins in previous issues of *Mathematics Teaching*, so I shall content myself by quoting: "Since mathematics is part of man's cultural heritage, we have no right to deprive anyone of the opportunity of sharing that heritage."

In a four or five year course we cannot hope to do more than introduce a child to a small part of mathematics and before drawing up a syllabus we must decide upon what basis we are going to select the topics we include. I suggest that, starting with a clear field, a syllabus might be framed bearing in mind Whitehead's "Mathematical Trinity: the variable, form and generality, which preside over the whole subject," interpreting these terms very loosely and remembering that the purpose is to provide a syllabus which will introduce the average child to the greatest glory of the human intellect. The aim will be to show something of the power of mathematical thought in relation to life around us, to give children a thorough appreciation of mathematical ideas and to develop the essential computational ability, bearing in mind that the only sound basis for efficiency in dealing with calculation is understanding of the concepts and techniques involved. We must also think in terms of the child's experience of mathematics and work through experiment and discussion to try to encourage close observation and a critical examination of a situation which will uncover the essential mathematics contained in it. There is general agreement that our pupils need constant reassurance that although mathematics is the language

of rational abstract thought it can also be used to describe everyday situations. Since the range of ability in Secondary Modern Schools is so wide, a single detailed syllabus for the whole school is somewhat unrealistic. It is, however, desirable to have a common framework around which courses for different streams can be built. This framework should be designed for the average or middle groups in the school and should be modified for the more or less-able groups. It is a great mistake to make the syllabus for the best children and cut it down to suit slower or less able pupils. The syllabus must be a servant, not a master, although it must be remembered that many teachers with little mathematical background and possibly little interest will tend to lean rather heavily on a written syllabus and, therefore, it must give them all the help possible. The actual form of a syllabus will vary greatly from school to school, depending upon the particular interests and experience of the available staff together with the local environment and any special interests of the school concerned, e.g. schools with a strong rural studies side or a large sailing club have scope for introducing specialised applications and perhaps some mathematical ideas usually thought too advanced for Secondary Modern pupils. In many cases the syllabus will be designed to include practical and social topics which should be chosen for the sake of their mathematical content and not merely for their own instrinsic interest.

Since 1945 there have been several series of textbooks written with Modern Schools in mind which include Arithmetic, Geometry and Algebra and sometimes a little Trigonometry, in an integrated whole. The approach to Algebra is usually the least successful part of these books. The textbook is only an aid and should not be allowed to dominate the course; many enthusiasts prefer to manage entirely without one. Choice of a textbook is a personal matter, but one should be chosen which most nearly fits the scheme designed for the circumstances of the school, and if possible other books should be available. Some textbooks are written for the children to read and learn about mathematics and its applications with little help from the teacher. Where it is necessary to work in small groups or individually these are very useful, but it must be borne in mind that Secondary Modern children usually find it very difficult to work in this way and indeed many may lack the necessary reading ability.

Often at our meetings we are asked "How do you test this sort of work?" as if the decision to adopt a new approach will depend on the answer. This, I think, is a relic of the days when arithmetic was looked upon as sums, testing and marking. Considered as an affair of rules and processes alone, mathematics is the mark-fiend's paradise. It is so simple to set a series of mechanical tests, easy to mark, objective, and productive of wonderful records of attainment and evidence of progress. Another drawback of the traditional objective test based almost entirely on mechanical manipulation is that it has a possible 100 per cent success. We see that some children have achieved a high score and we assume that if we go on long enough and strenuously enough teaching by the same methods, we shall raise the whole class to the same level.

A weekly test, a more formal monthly inquisition, together with termly (or sometimes half-termly) examinations quickly becomes a straight jacket in which our work is restricted and the supreme beauty of mathematics is lost. The need for testing our work is not denied, but the regular test is a fetish to be avoided; and we must not forget that it is our work as teachers which is being tested in the main. However,

there is room for imaginative testing which is in the spirit of our teaching; testing which springs naturally from our work, takes place at the most appropriate time, and is designed to find out how far we have failed to get the message over.

Many years ago A. S. Neill showed how the grasp of ideas could best be tested by presenting a graph of Church collections and asking the children to explain why the line went up and down. Much more can be learned from the answers to such a question than from graphs drawn when figures are given. If geometrical ideas have been taught through the medium of surveying, the test must be on the same lines. If children normally work in groups they should be tested on group work, even though this makes it very difficult to award each child his or her own mark. Children who learn through shopping activities can be tested in the same way. Even the most backward child should be asked to write a few lines describing an experiment or explaining a mathematical idea or process.

In many schools most classes of necessity will be taught by teachers whose first interest is not mathematics. Most of them will not have studied the subject beyond 'O' level and some of them either dislike the subject or are frightened by it. There is no reason why any conscientious teacher should not become a competent teacher of mathematics provided he or she is willing to be adventurous in approach, always trying to share the joy of discovery and the interest of the applications of mathematical concepts, and to learn from and with the children. Such teachers can find help in these books: *Teaching Mathematics in Secondary Schools* (Ministry Pamphlet No. 36); *Mathematics in Secondary Modern Schools* (Mathematical Association Report); *The Teaching of Modern School Mathematics* by F. J. James. There are others, but these three would provide a basis for further study. Attending A.T.A.M. Conferences and local Group meetings are also obvious steps to take. It is to be hoped that in every school there will be one member of the staff qualified or willing to become qualified to act as Head of Department and lead the team by co-ordinating the work of the school, and keeping in touch with work in other subjects and with local conditions to ensure that mathematics is not isolated but becomes a living study and an integral part of life both in and out of school. He or she must be prepared to devote a great deal of spare time to keeping abreast of developments taking place in other schools and adapting these ideas to suit his own school. He should be readily available for consultation by any member of the staff and prepared to advise on the selection and presentation of material.

Many schools have adopted the principle of re-grouping children from several forms of the same year into "sets" for mathematics so as to produce groups of roughly equal mathematical ability. Very occasionally there may be more sets than forms so that the subject gains by reduced numbers in the teaching unit. Setting throughout the school places a great strain on the timetable and accentuates the problem of staffing, even when the number of sets is the same as the number of forms. In the school in which I teach it has not been possible to provide complete setting at any time in the eight years or so since it was first attempted. We have usually had to accept a modification in the fourth year and occasionally in the third year as well. This table shows the distribution in our present second year which is fairly typical.

	Form				
Set	A	B	C	D	E
1	21	11	3		
2	13	15	4	2	
3	1	7	16	4	
4			5	17	3
5				2	10

It will be seen that the majority of children move only one group up or down, so that a great advantage is obtained even if only two or three forms can be time-tabled to take their mathematics at the same time. In large schools this may be the only possibility. The forms setted should be those containing the more able children; if taught by their class teacher, the least able are likely to benefit more from the continual contact with the same teacher, and the greater opportunity to integrate work in all subjects.

It is not easy to find a perfectly satisfactory method of allocating children to sets immediately on entry. The application of a formal test directly following a break of six or seven weeks with the children in unfamiliar surroundings and having come from different schools is unlikely to give satisfactory results. In our case selection is based on the scores obtained in standardised tests taken in the Primary schools, sometimes followed at the end of the first term by a further standardised test. In the main we rely on the teachers' ability to judge if a child is misplaced, and transfer as soon as possible so that by the end of the first year the sets are fairly homogeneous and are likely to be stable.

Setting may result in many more teachers having to teach mathematics against their own real inclinations and thus increase the number who teach the subject to only one class. With the shortage of mathematics specialists, even without setting, class teaching may be essential over part of the school. There is clearly much virtue in an organisation which provides continuity of teaching so that children do not have to re-adapt themselves at the beginning of each year, or even more frequently. It is not a good plan to attempt to overcome the shortage of specialists by splitting the work of each class so that some lessons are taken by a specialist and the rest by a class teacher; only in an emergency should a class be distributed in this way. It is probably during the latter part of a child's school life that continuity of teaching becomes essential, so that if non-specialist teaching cannot be avoided, it should be in the lower school, and thereafter as great a degree of specialisation and continuity as can be arranged. It is for the Head of Department to ensure that any discontinuity be reduced as far as possible. Every teacher should have a copy of the syllabus for the whole school and regular meetings of all teachers of mathematics and constant contact between the Head of Department and teachers of parallel sets should help to maintain a school pattern of approach and method. The best use of the Head of Department requires considerable thought; it will be advantageous if he has experience of teaching at all levels of age and ability in the school, while if he is the only member of staff who has been able to acquire a knowledge of mathematics well beyond the school syllabus it is clear that the more able pupils need his services toward the end of the course. It would be tragic if a whole group went right through the school without ever coming into contact with a teacher who approached the subject with enthusiasm and imagination. It follows that a further duty of the Head

of Department is to infect all other teachers with his own interests and enthusiasm. In a small school, it may be necessary to work in small groups within the class or even to plan for individual work.

If we have setting so that the most able mathematical pupils are together and a syllabus is followed which introduces Stage A Geometry and Algebra from the first, any children who are later transferred to Grammar or Technical Schools should not find themselves in difficulty. Presumably any children who stay at school to take G.C.E. will be in this group and they can be catered for together. No decision should be made before the end of the second year, and no formal G.C.E. work started until well into the third year. Provided that the first two years of the course have been full of experiment, experience and discovery, the formal work should give no difficulty.

The amount of time devoted to the subject will depend on the nature of the course planned and on local conditions; ideally five or six periods per week with at least one period of double length to make practical work or outdoor work possible and worth while. Geometrical drawing forms an integral part of the course from the first, but when it becomes necessary in the third or fourth year to develop this into Technical Drawing proper it will be found best to separate this from the mathematics course and provide for it on the time-table accordingly. This is done partly because the girls in a mixed school will not benefit so much as the boys from such a course, and also since the separation allows the staff of the technical subjects a clearer field to make use of this training; it may be desirable to regard this as part of the wood- and metal-work side of the school, though not necessarily taught by the craft teachers.

The atmosphere in which mathematics is studied can have a tremendous effect and whenever possible a room or rooms should be set aside for the purpose. Clearly it is not possible for every child to have all his mathematical lessons in a specialist room; some may be done, to mutual advantage, in other specialist rooms e.g., practical rooms, science labs., geography room. Alert teaching can make use of whatever environment is available. A specialist mathematics room should be on the ground floor opening onto the playing field. It needs ample wall space for display, easily movable furniture, tables rather than desks, possibly a bench, and ample cupboards and shelves. It must be equipped for projecting films and filmstrips. A large expanse of blackboard, including a squared board is essential. The possibilities for equipping such a room are tremendous, but although expensive precision instruments are attractive they are not essential and simple school-made apparatus is usually more valuable. The making of apparatus gives a valuable insight into the mathematical principles involved and it is worth while to design models which can be made and used once and then thrown away or taken home. It is not fair to try to save time by asking this year's class to use the instruments made by last year's. Clinometers and angle measures, Jacob's staff, etc., can be improvised in cardboard; parts can be prefabricated beforehand. We require an adequate supply of all forms of measuring apparatus, metric as well as imperial units, simple surveying apparatus, and models of solids in card, wood or wire. If the course is to include some applied mathematics, then material for the construction of large scale experiments should be added, and the room fitted with a beam from which heavy weights can be hung. We must have pulley blocks of all sizes, plumb bobs, stop watches, crow

bars, and a steelyard, etc. Any mechanisms which can be obtained, old clocks, milometers, and so on which can provide a basis for a mathematics lesson should be included. In this room the children should be able to see apparatus and work in progress illustrating all parts of the course so that they can see where they stand in relation to the course as a whole. There should be equipment of a robust nature left about for personal experiment; a few Geo-boards and rubber bands are most inviting. If possible children should be encouraged to come into the room out of lesson times and make their own models and experiments. There should be a library always available.

From the very beginning homework should be set. Apart from any other reason it ensures that parents take some sort of an interest in the child's work. Homework may take different forms: collection of material for class discussion later; statistical data collection; examples of the use of geometrical shapes; and similar work. The completion of work started in school time may be suitable homework. In general, particularly in the early years, drill exercises of the traditional pattern should be avoided, with the possible exception of drill on work which is well known but may not have been used in school recently.

In conclusion may I say that despite the dozen or so years that have passed since the Secondary Modern Schools were created, mathematics is still in the experimental stage in these schools, and we must see that it becomes a study in its own right and not a watered-down version of Grammar or Technical school mathematics.

(In our next issue there will be articles on the first year in the Secondary Modern School and on running a Mathematics Club in the School).

SIMPLE MODEL MAKING IN PLASTIC

I. B. REYNOLDS

The use of plastics in model making has many advantages which hitherto have been denied the modeller using wood, metal or card. It is cheap, always clean, light, durable and is easily worked, but its greatest asset is that it is transparent, lending itself to such examples as are given below.

Cellulose acetate sheet 20 thousandths of an inch thick is available from craft shops and from branches of Halfords Ltd., at a cost of about one shilling per square foot, and acetone, the solvent for welding the joints, can be obtained from any school laboratory.

By the use of a sharp point of a pair of dividers, any shape or figure can be scored on the surface of the sheet, and by gently bending along the score, the figure can be obtained intact. When solids are to be built in this material, the edges of each figure should be smoothed and mitred with the aid of glass paper. In order to effect a weld between two surfaces, the two edges to be joined should be placed together, as in Figure 1, and held in position with Sellotape. Bend the two surfaces to the required angle, as in Figure 2, and using a glass rod previously dipped in acetone, transfer a 'run' of acetone along the edge. Hold the two surfaces together

FIG. 1.

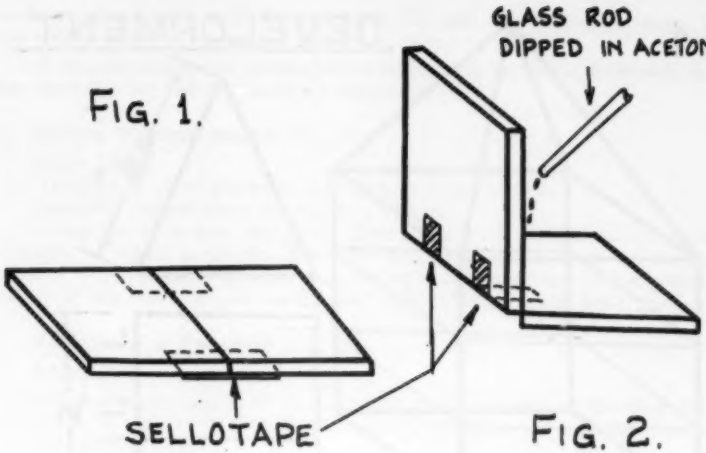
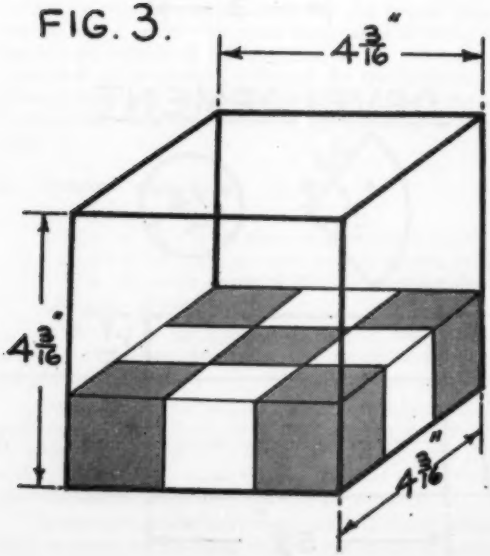


FIG. 2.

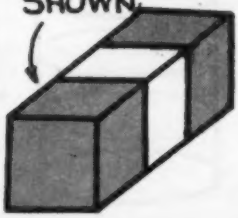
FIG. 3.



WOODEN BLOCKS

$4\frac{1}{8}" \times 1\frac{3}{8}" \times 1\frac{3}{8}"$

COLOURED INTO "CUBES" AS SHOWN.



bars, and a steelyard, etc. Any mechanisms which can be obtained, old clocks, milometers, and so on which can provide a basis for a mathematics lesson should be included. In this room the children should be able to see apparatus and work in progress illustrating all parts of the course so that they can see where they stand in relation to the course as a whole. There should be equipment of a robust nature left about for personal experiment; a few Geo-boards and rubber bands are most inviting. If possible children should be encouraged to come into the room out of lesson times and make their own models and experiments. There should be a library always available.

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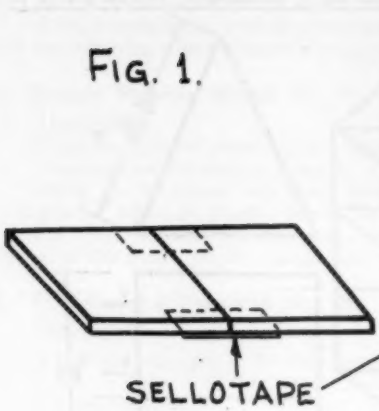


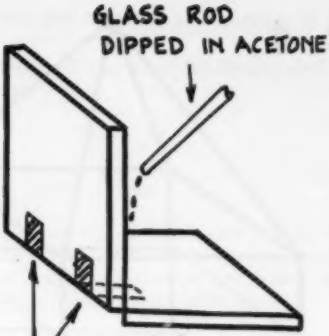
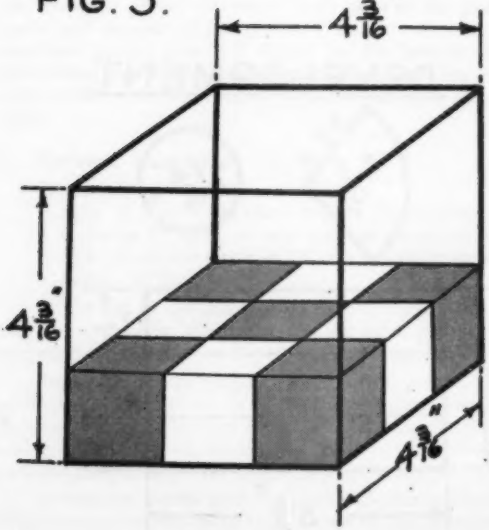


FIG. 2.

FIG. 3.



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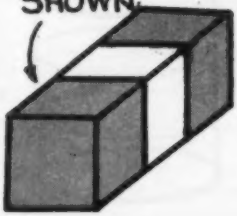
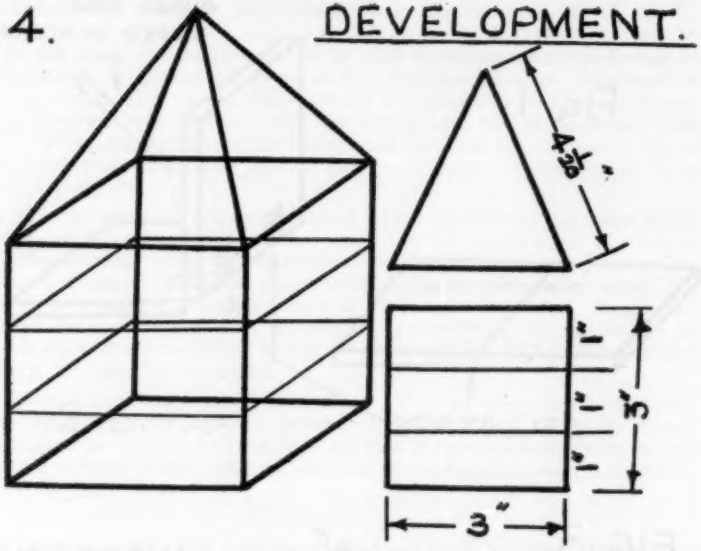


FIG. 4.



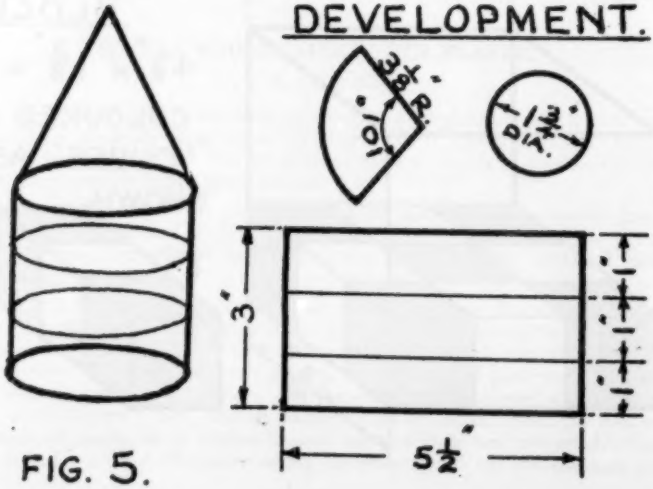


FIG. 5.

at the required angle until the acetone is dry and remove the Sellotape. A strong join will be found to have been made.

A little experience in working this medium will pay great dividends, and therefore the following order of models is suggested.

1. Simple Volume Model (Fig. 3)

Materials:

9 lengths of wood planed to size $4\frac{1}{8}$in. × $1\frac{3}{8}$in. × $1\frac{3}{8}$in.

5 squares acetate sheet size $4\frac{3}{16}$in. × $4\frac{3}{16}$in.

Using the procedure explained above, construct an open ended cube $4\frac{3}{16}$in. × $4\frac{3}{16}$in. × $4\frac{3}{16}$in. in plastic. Mark the lengths of wood, as in the diagram, into cubes of $1\frac{3}{8}$in. (without actually constructing cubes) and place in the plastic model. By use of this model pupils can see how 'volume' is gradually built up.

2. Volume of a Pyramid (Fig. 4)

Materials:

4 Isosceles triangles in 20 thou. acetate sheet as shown in 'Development.'

4 3in. × 3in. × 20 thou. acetate sheet, scored at one inch intervals as shown in 'Development.'

1 3in. × 3in. × 20 thou. acetate sheet, not scored, for the base.

Quantity of fine building sand.

As in the simple volume model, construct an open ended cube 3in. × 3in. × 3in. in plastic, making sure that the scored lines coincide and the base is plain. On the open ended top erect three sides of the pyramid and seal all joints on the inside and outside of the model. Pour into the model sufficient sand to fill exactly one-third of the cube (as shown by the bottom scored line). Seal the model by positioning the final isosceles triangle. Invert the model and the sand will clearly demonstrate that the volume of the pyramid is exactly one-third the volume of the cube.

3. Volume of a cone. (Fig. 5)

Materials:

1 $5\frac{1}{4}$in. × 3in. × 20 thou. acetate sheet scored at one inch intervals.

1 sector in 20 thou. acetate sheet as shown in 'Development'.

1 $1\frac{3}{4}$in. diameter circle in 20 thou. acetate sheet as shown in 'Development'.

Quantity of fine building sand.

If the aid of the Handicraft Master can be enlisted, two formers of wood, one a cylinder of $1\frac{11}{16}$in. diameter, and the other a cone of $1\frac{11}{16}$in. base diameter and 3in. perpendicular height, will prove of great assistance.

Immerse the rectangle and sector of acetate sheet in warm water and, when malleable, mould them to adopt their shapes (using the formers if available). Hold in position with Sellotape until cold and weld the joints with acetone. Weld the cone and cylinder together, fill the cone exactly with sand, and weld on the $1\frac{3}{4}$in. diameter circular disc to form the base of the cylinder.

Invert the model and the sand will clearly demonstrate that the volume of the cone is equal to one-third the volume of the cylinder.

SERIES

DAVID H. WHEELER

The study of sequences in school mathematics (when dealt with at all) is often confined to Arithmetic and Geometric Progressions and left until late in the course. Some texts make small concessions to a more general treatment in an introductory paragraph to a chapter dealing with Progressions, but rarely take the opportunity to draw examples from situations within the experience of the pupils. It seems a pity not to use the fascination and appeal of an ordered series of numbers at an earlier stage. The ability to "spot" the rule of formation gives great satisfaction, and if a preliminary study is anchored firmly to examples deriving from suitable situations, a surprising degree of skill in generalisation and justification can be developed.

In order to illustrate this possibility, there follows a sketch of an approach to a particular series, one of several that arise naturally in work that can be undertaken with pupils of twelve years and upwards. The suggestions here are not intended to be exhaustive but illustrative, and with a little thought, other, equally suitable tactics can be devised. In any case the rate and extent of the investigations will depend on the ability of the children to find for themselves the significant results, and they will almost certainly expose for discussion several facets that have been overlooked by the teacher. The degree of formalisation of the work will again depend on the children. It should not be demanded that everyone produce a full algebraic proof of the properties discovered, though the abler pupils will certainly wish to do so.

The starting point of the investigation is a discussion of the maximum number of intersections that are produced when a specified number of straight lines are drawn in a plane. The investigation is at first experimental. Drawings of 3, 4, 5, 6 and-so-on lines are made, and the intersections counted. (This in itself raises points of interest, and several profitable lines of enquiry are suggested by the children. What happens if two or more lines are parallel? If three or more are concurrent? Is it possible to draw a given number of lines so as to intersect in a given number of points, less than the maximum possible? Is it always possible to illustrate all the intersections of, say, fifteen lines on a limited piece of paper?) As the number of lines increases, and the possibility of error in totalling the intersections increases, it is found that some of the children find their own answer to the question for the subsequent number of lines without drawing, and have considerable confidence in their deductions. This is the time to suggest an orderly tabulation of the results so that all the children can see what some have already found. On the production of a list of the answers, many more will continue the table correctly without further assistance, and a discussion can take place on the methods used to arrive at the subsequent entries.

No. of lines	*Intersections*
1	0
2	1
3	3
4	6
5	10
6	15
7	21
etc.	etc.

The explanations of the methods used to extrapolate will vary, although some will be seen to be equivalent. For example, some pupils argue from the successively increasing differences in the numbers of intersections, and are not aware of using the first column numbers at all. Others will add across each row to obtain the next second column entry—probably the commonest reaction—and others still will see the second column entries as continued sums of the integers from the first

column. It is rare, unless some similar situation has already been investigated, for the device of halving the product of consecutive entries in the first column to be discovered. This rule is, of course, the most useful for a quick algebraic generalisation, and it is worth presenting an allied situation which makes this discovery more apparent. (To make the pupils *want* a generalisation it is only necessary to ask for the maximum number of intersections of 100 lines—that is, if some bright pupil has not already chucked that particular spanner).

Before passing on, though, it is worth pointing out that *all* of the methods of continuation adopted are valuable and give additional insight into the properties of the sequence. Also, an attempt should be made to face the question that now arises, which is: How do these numerical devices correspond to the situation that gave rise to the table in the first place? This is important in stressing the *continuity* of the situation as the number of lines increases. Originally, the drawings were quite separate investigations in which previous results were not assumed. But the extrapolation of results in the table implies a dependence on previous information, and it is now clear that a better approach to the initial drawing situation would be to *add one line at a time* to the same figure. With most pupils, a recapitulation of this kind, will lead to a realisation of the legitimacy of their methods of continuing the series. (They see, for instance, that the addition of an extra line, which will cross all of the existing lines once each, adds that number of intersections to the number previously obtaining).

If the pupils now compile a similar table for the intersections of coplanar circles, they are pleased to notice an important similarity of results.

No. of circles	*Intersections*
1	0
2	2
3	6
4	12
5	20
6	30
7	42
etc.	etc.

They see that the number of intersections is, in each case, twice the corresponding number in the preceding table, and it is not difficult for them to see why. Also, when continuing the entries, they very quickly see that multiplication of each successive pair of integers from the first column provides them with their second column figures. So that now, they can find the maximum number of intersections of 100 circles without difficulty, and hence the number for 100 lines, which they may not have been able to do before. The children now know, whether or not they have formulated it algebraically, that the number of intersections of n circles is $n(n-1)$ and that the number of intersections for n lines is $\frac{1}{2}n(n-1)$. In classes which have a reasonable facility with algebraic notation, this formulation does not seem a further difficulty, but a useful summary.

Again, it is valuable to try to find the justification for the formula $\frac{1}{2}n(n-1)$ from the original situation, and with a little assistance the pupils probably can. If there are n intersecting lines, there are $(n-1)$ intersections on each line. The product $n(n-1)$ counts each intersection twice because each intersection lies on two lines. So the total number is $\frac{1}{2}n(n-1)$.

The series 0, 1, 3, 6, 10, 15, etc. which has been discussed can easily be produced by considering other situations. For example, let the pupils take a given number of different objects—say four differently coloured Cuisenaire rods—and find how many different pairs can be formed from them. In the case of four rods, pairs can be abstracted in six ways. The children repeat the experiment with other numbers

of objects, and it will not be long before they systematise their discoveries and find the same sequence. Depending on the age and the ability of the pupils, a more or less formal expression of the reason can be reached. (It will be noticed that this generation of the sequence is not mathematically distinct from the preceding case).

A situation in which the result is less obviously a consequence of similar processes is to take a pair of identical objects—two yellow rods, perhaps—and consider in how many ways they can be disposed in a given number of boxes. Suppose that there are three boxes and a zero is used to denote an empty box. Then the possibilities are: (2,0,0), (0,2,0), (0,0,2), (1,1,0), (1,0,1) and (0,1,1), i.e. a total of six ways. (The boxes are regarded as ordered; they can be labelled A, B, C). Repetition of the procedure with varying numbers of boxes produces the table:

No. of boxes	*No. of ways*
1	1
2	3
3	6
4	10
5	15
etc.	etc.

This is, of course, the same series, although displaced in position relative to the value of n. The general term is now $\frac{1}{2}n(n+1)$. Why?

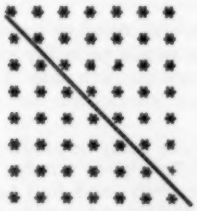
Three ways of deriving the sequence 0, 1, 3, 6, 10, etc. have been given and each one is capable of practical verification for small values of n. At the end of some experience along the lines described, the pupils will have a variety of facts and impressions, all of which will be valuable in later work.

Further study of the numbers in the series can be made. If the children have met Pythagorean triangular numbers, they will recognise that the terms of the series are triangular.

1 3 6 10 15 etc.

What has already been discovered indicates that the generating function of these triangular numbers must be $\frac{1}{2}n(n+1)$ where n is the number or rows in the number. Why should this be so? Pupils of thirteen or fourteen can produce quite elegant demonstrations.

Suppose the triangular configurations are wrenched into right-angled isosceles form. Consider an n-row number.



This can be combined with a duplicate turned through two right-angles to form a rectangular number with sides n and $(n+1)$. Since the number in the rectangle is $n(n+1)$, it follows that the triangular number is $\frac{1}{2}n(n+1)$.

Or, a square number, of side n, is twice the corresponding triangular number, except for a difficulty with the diagonal. This can be corrected by adding on a spare diagonal before halving—giving $\frac{1}{2}(n^2+n)$—or adding on half a diagonal after halving the square—i.e. $\frac{1}{2}n^2+\frac{1}{2}n$. Alternatively, the number can be found by totalling the columns of which it is composed. One way is to consider the sum as

equivalent to n columns with an average number of $\frac{1}{2}(n+1)$ in each (because the columns can be grouped in such a way that each pair has an average equal to that of the average of the first and last).

Many more properties of this interesting series will be discovered at different stages in the school. No mention has yet been made of the fact that consecutive pairs add to give a square, or that the sequence will be found as a diagonal of Pascal's Triangle. At Sixth Form level, the numbers appear as the coefficients in the expansion of $(1 - x)^{-3}$.

If the series is to be summed, then it is worth looking at the original drawings of intersecting lines and counting the number of triangles formed. With 3 lines there is 1 triangle, 4 lines yield 4 triangles, 5 lines yield 10 triangles, 6 lines 20 triangles, and so on. The sequence obtained (1, 4, 10, 20, 35, etc.) contains the progressive sums of the original series. With some difficulty, a formula can be derived for the new series which is equivalent to a summation formula of the first.

Enough has been said, not to provide a blue-print for a few easy lessons, but to provoke thought and experimentation in this rich field.

GROUP MEETINGS.

CAMBRIDGE AREA GROUP.

More than seventy teachers of mathematics in Infants, Primary and Secondary schools within a thirty mile radius of Cambridge gathered at the Swavesey Village College on Saturday, 27th June, to attend a Conference entitled *Consideration of the Teaching of Mathematics in Primary and Secondary Schools*. Miss J. Clarkson spoke on "The Material Usage of Cuisenaire Rods in the Primary School," and Mr. R. H. Collins on "A new outlook in the Teaching of Mathematics." The meeting was very successful and one important outcome was the decision to form a new local group of A.T.A.M. to be known as the *Cambridge Area Group*.

The next meeting of the Group will be held on Saturday, 28th November, 1959, at the Grammar School for Boys, Queen Edith's Way, Cambridge, when topics to be discussed will include "The Primary School Syllabus" and "The Third Year Course in Mathematics." Details of this meeting and information about the Group may be obtained from the Secretary, Mr. N. Reed, the Village College, Swavesey, Cambridge.

NORTH-WESTERN GROUP.

The Group held its Autumn Term meeting at the School of Education, Manchester, on Saturday, October 10th, when Mr. E. T. Norris of Saltley College, Birmingham, gave a stimulating talk on "The Function Concept." Earlier the Group had discussed the Ministry's Report on "Teaching Mathematics in Secondary Schools" and the Mathematical Association's Report: "Mathematics in Secondary Modern Schools."

NUMBER AND INFANTS

by R. M. Fyfe

What is Learning?

Educational psychologists have been known to say to teachers that the concept comes first. But in actual fact, the concept comes somewhere in the middle of an act of learning.

At first there is the feeling of being surrounded by undifferentiated sensations: some of these do impinge on us from without (given by the five senses) but they include our inner physical, emotional and mental happenings.

In a stable situation, the sensations group themselves, because we are "attending", and then we have what can be called perception. This is a stage of self-organisation in which we "know what we are looking at".

The field contracts once more and, often suddenly, we think we see one "thing". This is the moment of intuitive awareness of a concept.

The concept is not yet clear: to make it so we need to express it—to pin it down. This requires symbols, which are spoken or written *words*, diagrams, marks on paper (or on the blackboard) which are accepted as "models" which represent the idea we have in mind. The purpose of the symbolising is twofold: to clarify the idea, and to communicate it. When the concept has in some such way been clarified, it becomes usable.

The stage of techniques begins. The writings are elaborated and given form and shape. If the learning is of mathematics, we are now at the stage of "doing sums". This is not an end in itself; it is a further process, the aim and object of which is mastery of the idea and gain of power which mastery gives to the mind and being of the human person who thus engages himself.

It is only necessary to add that children do all this for themselves. Each one of them *has* to do it for himself, for it is an inner development.

How can the Teacher Help?

Three basic assumptions are possible:

1. that children have minds which they do and must use (for survival), and that they enjoy using their minds creatively,
2. that they like making and doing things with their bodies, particularly with their hands,
3. that there is a social factor in that the approval or disapproval of the other members of the group (and this includes the teacher) matters.

Work and Play

The distinction between the two is arbitrary. Play is the absorbed attention in what interests a child, and from it, inevitably, much learning takes place. We cannot *want* work to be less than this, although we have to admit that for many of us it is.

Is it possible for the love of living which is manifested in a child's play to extend into and stay in all the activities of the school?

Apparatus

This can be classified under two headings, although there really is no clear-cut division. We can say that some of the material used is mainly to develop knowledge of what is used in the world of practical life, e.g. money, measuring, weighing; whereas other objects are a means for activating thought, for exploring the world of ideas.

Apparatus of either kind is intended also to provide the emotive stimulus, a field of interest.

In conclusion, I want to explain the merit of Cuisenaire rods as a material which can make number concepts fully mathematical.

Cuisenaire Rods

Tip the contents of a box on to a table and invite play. Everyone becomes absorbed in doing something. So we are sure that we have provided a "field of interest".

Next we notice that relational thinking is taking place. Some rods are *greater than* others (or *less than*), others are *equal*. Every rod is a rod, an element of the set of all the rods of the boxful. These facts are made use of in the patterns we see being made and if one desired rod has "run out", others are substituted. The rods may be used to *represent* objects (houses, ships etc.) and each child is creating his own symbols of this. Although a piece of wood is a piece of wood, it is also a door, or a man, or a bridge.

When we begin to conduct a "lesson" and to ask questions, all the children tell us that if we put two rods together end-to-end

that we are *adding* them. Then if we find the third rod which provides the *answer* to this addition, we have a pattern of three rods which can be read in different ways, and we "see" the commutative principle: $a + b = b + a$. If one rod is removed from the pattern, the other two show a *difference* which can be thought of as *subtraction:* either short rod can be subtracted from the long one to give the other short one. Moreover we have here a *group* in which addition and subtraction are clearly an "inverse pair of operations"—part and parcel of the one situation.

From an extension of "end-to-end" we can find the decompositions of any rod, keeping all that is possible, and rejecting what is impossible. In particular we can notice that some rods can be decomposed into rods of one colour at a time in several ways. These are *factors*. Each factor is a fraction of the rod from which we started. From this, with the help of the unit rod (white) we can find that every rod is some fraction of every other rod.

What is a number?

If we are thinking of integers (whole numbers) we have to refer to a unit which measures it. But there are an infinite number of units. We measure number of apples in terms of *one* apple, or in terms of a *dozen* apples, or it could be in terms of *scores* or *hundreds* or *boxfuls*.

If the red rod (2 white rods end to end) is the unit, then crimson is 2 and brown is 4 and white is $\frac{1}{2}$. If crimson is the unit, then brown is 2, red is $\frac{1}{2}$ and white is $\frac{1}{4}$. What is $\frac{3}{4}$ in this case? It is a light-green rod thought of as being associated with the crimson rod which is the measuring unit.

Returning to factors, we see that the two crimson and four red rods, which both make decomposition lines of the brown rod, can be put side-by-side to make *rectangles* of the same shape and size.

They are *superimposable*, i.e., they fit on top of each other. So we can say that a crimson rod *measures* the number of red rods (4), and that the red rod measures the number of crimson rods (2). From this a new rod symbol arises (for multiplication). A cross of a crimson rod on top of a red rod means "four reds". We can think of the crimson rod as being an indicator or operator, and we know that we are really talking about a line of red rods (four of them) placed end to end. Alternatively, a cross formed by placing a red rod crosswise on top of a crimson rod means "two crimsons". The commutative principle is again clear and evident; $a \times b = b \times a$.

At every stage of the work conventional writings can be introduced, but also there is a rod meaning. Children can and do return to the rods to help themselves with their mental work, of which the writings are (and should be) only symbolic explanations and descriptions.

Contrary to what some teachers think, *all* arithmetic is mental. For example, to add figures standing for numbers is impossible; what we do is to add the numbers for which the figures stand. We do this adding in our minds and then write down the result with other figures. How can the marks on paper $7 + 5$ produce 12 as "answer"? The aim of all our number work is to help the child to create a mental store of correct answers. It has been proved that the bridge of visualising a pattern of a black rod end to end with a yellow rod (together with the orange-red which gives the answer) is a quicker way towards memorising the desired *fact*, than going through the laborious counting of $7 + 1 + 1 + 1 + 1 + 1$ many many times, and also having $5 + 1 + 1 + 1 + 1 + 1 + 1 + 1$ as a dissociated and "more difficult" different sum. As mentioned above, the rod method also links into this group $12 - 7 = 5$ and $12 - 5 = 7$.

SPACE IN CONGRESS.

The committee (of Congress) were in some geometrical difficulty about what would happen if national boundaries were extended into space. Senator Keating (Republican, New York) argued that, because of the curved surface of the earth, extended boundaries, in the form of an "inverted cone," would overlap, with the result that more than one nation would be occupying the same air space.

Another member felt, on the contrary, that, if boundaries were projected upwards like columns, gaps would be left between them. But, asked Senator Keating, would not the rotation of the earth cause the columns to get mixed up?

A Republican colleague gave him "the full weight of my scientific ignorance," and the committee was about to vote on "gaps or overlaps" when someone thought of approaching the National Geographic Society for a ruling.

(Report in the *Times;* 7.3.59).

NEW APPARATUS AND MATERIALS

MATHEMATICAL PIE MATERIAL

We have received from *Mathematical Pie* a copy of their new catalogue which gives full details of the range of publications and materials which they now produce or act as agents. It is hoped to review these from time to time in our columns; at the moment mention may be made of various productions which have made their appearance in the past twelve months.

In addition to *Mathematical Pie*, the Trust now handle *Le Facteur X* (published nine times yearly), *The Mathematics Students Journal* (published four times yearly from the U.S.A.) and *Topic* (a new magazine intended as an aid to general studies in the Sixth Form). *Nomograms* is a 24-page booklet giving an introduction to this topic at secondary school level.

Two new film-strips are *Alice in Numberland* and *Calculoting Machines*, and these are available on free loan for preview purposes, as are the other Mathematical Pie film-strips. Finally, two new wall charts are *The Beginning of Astronomy No.* 2, and *The History of Units, No.* 1, and the Trust are now handling the import of the portfolio of portraits of mathematicians produced by Scripta Mathematica.

Full details of these and all other Mathematical Pie Publications and Teaching Material are contained in the Catalogue which is obtainable free of charge from Mathematical Pie Ltd., 97, Chequer Road, Doncaster.

MATHEMATICS TEACHING PAMPHLETS.

The first of a new series of pamphlets produced by the Association for Teaching Aids in Mathematics under the title *Mathematics Teaching Pamphlets* have just been published. Their purpose is to assist teachers in the selection, preparation and use of aids in teaching.

Pamphlet No. 1 is a revised edition of the *Film and Filmstrip List* originally published by the Association some years ago. The list gives full details of all mathematical films and filmstrips at present available, with particulars of sources, prices, etc. It should prove invaluable to all teachers of mathematics.

Working Model Wallcharts (Pamphlet No. 2) gives practical details for the construction of wallcharts for the teaching of geometry. The charts may be constructed quite easily from the instructions given in the pamphlet, and once the technique has been appreciated it should be a simple matter to construct from sheet cardboard and elastic the various models described in the pamphlet and even proceed to develop one's own ideas for these useful teaching aids.

Pamphlet No. 3 (*Looking at Rhombuses*) and Pamphlet No. 4 (*Regular Hexagon*) both deal with the development of ideas on these two topics using geo-boards. The possibilities of the material are clearly brought out, and once again the pamphlets not only give adequate guidance to cover the particular points of the pamphlet, but provide an inspiration for further development through one's own ideas.

Pamphlets Nos. 1 and 2 cost one shilling each, and Pamphlets 3 and 4 are sixpence each. Copies are on sale at meetings of the Association, and may also be obtained from the Secretary of the Association, 318, Victoria Park Road, Leicester (please include twopence extra for postage).

STRUCTURAL ARITHMETIC MATERIAL.

Most of our readers are fully aware of the need for using concrete material in the teaching of elementary number processes and the latest material to be made available in this country is Dr. Catherine Stern's *Structural Arithmetic*. It has been used in the United States for some years and is now being produced in this country by the Educational Supply Association Ltd.

Dr. Stern has devised a series of models (cubes, blocks, counting and pattern boards, number cases and number track) to represent number relationships and to demonstrate the arithmetical processes met by children during the Primary School stage or in remedial teaching of older children.

Through the courtesy of the Educational Supply Association we have been able to examine the Structural Arithmetic material and at present we are trying it out under classroom conditions; we hope to publish a full review in our next issue. Meanwhile full details of the material are available from E.S.A., Dept. S.A.4, 181, High Holborn, London W.C.1., and a full account of the principles underlying the material and the methods of use is contained in *Children Discover Arithmetic* by Dr. Stern, published in this country by G. G. Harrap & Co. Ltd.

FOREIGN PUBLICATIONS

April saw the first appearance of *Praxis der Mathematik*, a new monthly magazine devoted to the teaching of pure and applied mathematics. Edited by Dr. G. Wolff, of Dusseldorf, the journal is published by Aulis Verlag Deubner and Co. of 6/12 Antwerpener Strasse, Cologne, West Germany.

The twenty-eight page first number contains articles on number theory, descriptive geometry, nomograms and logarithmic and exponential functions as well as sections devoted to news and reviews. Particularly interesting is the attention which the journal is giving to the encouragement of examples in applied mathematics which are genuine, and not merely exercises in pure mathematics. Finding examples of this kind and keeping them up to date is one of the greatest "problems of problems," and we will watch developments here eagerly.

The first number of *L'Enseignement des Sciences* has just appeared. A very strong editorial committee is producing an impressive periodical which will undoubtedly make a great contribution to the teaching of science and mathematics if the high standard of the first number is maintained.

Mathematicians will be particualrly interested in the articles by Choquet, *Les Mathematiques modernes et l'enseignement*, and Jeronnez, *Qu'est-ce que la geometrie intuitive*? The former discusses some of the principal ideas of modern mathematics and considers their repercussions on more elementary teaching, and the latter describes some of the recent developments in the teaching of geometry in Belgium. How much further evidence is needed before English teachers realise that our Victorian pre-eminence is no longer maintained, and in spite of Zeta and jet propulsion we live in what is in some ways a backward country?

L'enseignement des Sciences is well produced, in pleasing format with excellent illustrations. It is a pity that we have nothing as good over here. It appears every two months, and the address is 115 Boulevard Saint-Germain, Paris VI.

STRUCTURAL

Dr. Catherine Stern's

ARITHMETIC

The ESA are pleased to announce that they are now manufacturing and distributing this important material.

STRUCTURAL ARITHMETIC can assist teachers by representing in concrete form all the arithmetical processes taught in the Primary school and in remedial teaching of older children.

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A STROBOSCOPIC DEMONSTRATION OF WAVE MOTION

by R. B. Morrison

During a recent holiday lecture to children on waves and vibrations, the author required some way of demonstrating the principles of longitudinal and transverse wave motion on a large scale. The small demonstration models used in the classroom are quite inadequate for a large lecture hall. The method finally adopted was a modification of the simple stroboscopic disc and a description of the construction of such a disc may be of interest to those readers who find themselves in a similar position.

The principle of simple harmonic wave motion is that each particle of the medium executes simple harmonic motion about a zero point, while adjacent particles execute a smiliar motion but with a slight change in phase. The distribution of phase along the particles is sinusoidal so that after a distance equal to the wavelength a particle in phase with the original is reached. In the case of transverse

wave motion the direction of the simple harmonic motion is at right angles to the direction of propagation of the wave, while for longitudinal wave motion it is in the direction of propagation. In both cases the motion and the phase may be represented by a sinusoidal function. Modulation of the simple stroboscope with a sinusoidal displacement of the strobe lines either at right angles or parallel to their spacing on the circumference of the disc is all that is required.

The prototype model consisted of a wheel one metre in diameter divided into three concentric zones. The inner zone consisted of the ordinary stroboscope with 60 black strips about $\frac{1}{2}''$ wide and $2''$ long spaced at 6° intervals round the zone. The regular spaced lines from the centre of the wheel radiating through these strips were continued into the outer two zones to serve as reference lines for the subsequent modulation. The outer zone was modulated for longitudinal waves and the middle zone for transverse waves.

It was decided to include three complete waves round the circumferal zones, so that each cycle occupied 120° of arc and the modulation was then repeated for the remaining two cycles. This meant that 20 strips at 6° intervals had to be fixed according to the sinusoidal distribution, the last strip being in phase with the first and back on its reference line. The maximum displacement of any strip may be chosen for convenience. In the case of the longitudinal pattern the maximum angular displacement of each strip from its reference line was 5°. For the transverse pattern it was 10 cm. along the radius. A table was constructed to give the positions of each strip in a cycle.

As the cycle was to repeat every 120° and the maximum displacement for the longitudinal pattern was 5°, the displacement for a strip making an angle θ with the first strip may be represented by the equation:—

$$y = 5 \sin 360.\ \theta/120. = 5 \sin. 3\theta.$$

A continuous function is not plotted so that values of θ are taken at 6° intervals to give the displacement at each of the 20 reference lines in the cycle. The calculated displacements and positions of the strips are given in the table.

MODULATION TABLE FOR LONGITUDINAL WAVES

θ	sin 3θ	5 sin 3θ	Position (°)
0	0·0000	0·0000	0·0000
6	0·3090	1·5450	7·5450
12	0·5878	2·9390	14·9390
18	0·8090	4·0450	22·0450
24	0·9511	4·7555	28·7555
30	1·0000	5·0000	35·0000
36	0·9511	4·7555	40·7555
42	0·8090	4·0450	46·0450
48	0·5878	2·9390	50·9390
54	0·3090	1·5450	55·5450
60	0·0000	0·0000	60·0000
66	— 0·3090	— 1·5450	64·4550
72	— 0·5878	— 2·9390	69·0610
78	— 0·8090	— 4·0450	73·9550
84	— 0·9511	— 4·7555	79·2445
90	— 1·0000	— 5·0000	85·0000
96	— 0·9511	— 4·7555	91·2445
102	— 0·8090	— 4·0450	97·9550
108	— 0·5878	— 2·9390	105·0610
114	— 0·3090	— 1·5450	112·4550
120	0·0000	0·0000	120·0000

The black strips in the outer zone are placed at the positions indicated in the final column. Clearly they cannot be placed to the accuracy indicated by the sine tables, and it may be found more convenient to convert the angular displacements to lengths of arc at the position of the mid-zone. Thus for the prototype a 6° angle corresponded to about 4 cm. at the longitudinal zone.

The modulation for the tranverse wave zone is similar, except that the displacements are radial from the mean circumference of the zone. In this case the angular separation of the strips is constant at 6° intervals but their radial displacement follows a sinusoidal distribution. For a maximum displacement of 10 cm. the displacement of each strip in one cycle is given by:—

$$y = 10 \sin 3\theta$$

The radius of the mean line for the transverse zone was chosen as 20 cm. and the radial distances to the centre of each black strip in a cycle calculated as before.

MODULATION TABLE FOR TRANSVERSE WAVES

θ	10 sin 3θ	Radius (cm)	θ	10 sin 3θ	Radius (cm)
0	0·0000	20·00	66	— 3·090	16·91
6	3·090	23·09	72	— 5·878	14·12
12	5·878	25·88	78	— 8·090	11·91
18	8·090	28·09	84	— 9·511	10·49
24	9·511	29·51	90	— 10·000	10·00
30	10·000	30·00	96	— 9·511	10·49
36	9·511	29·51	102	— 8·090	11·91
42	8·090	28·09	108	— 5·878	14·12
48	5·878	25·88	114	— 3·090	16·91
54	3·090	23·09	120	0·000	20·00
60	0·000	20·00			

The sinusoidal modulation for the transverse waves must needs be somewhat distorted due to the divergence of the radial lines. Nevertheless the wave motion can be followed as it moves round the zone.

The completed wheel is rotated by a motor such that the speed of rotation may be varied, and the surface of the wheel illuminated with an A.C. source of light. For the conventional A.C. mains with a frequency of 50 cycles sec $^{-1}$, the wheel face will be illuminated once every one-hundredth of a second. The stroboscopic effect will therefore be seen when the speed of rotation is such that the wheel turns through 6° in this time, i.e. a speed of rotation of 100 revolutions per minute. In practice the speed is adjusted until the inner strobe pattern is seen to be stationary when both the longitudinal and transverse wave motions will be observed in the outer zones travelling round the surface of the disc. The regions of compression and rarefaction can be seen quite clearly in the former case.

A simultaneous demonstration of both types of wave motion in the manner described above, together with the similarity in construction, illustrates to a class the basic fundamentals of such motion and how it is related to simple harmonic vibrations. In addition pupils may construct their own wave strobes as an exercise based on the gramophone speed of rotation of 78 revolutions per minute. In this case it may be found more convenient to confine each type of wave motion to a single strobe.

MATHEMATICS IN SECONDARY MODERN SCHOOLS.

Mathematical Association Report.

The secondary modern school is in process of crystallising its curriculm after an existence of only 15 years. Like all processes of growth it began slowly. The simplest features of the old elementary schools were used as a starting point and many children received an unrestricted, if somewhat repetitive, diet of commercial arithmetic, a repertoire of tricks, methods and catches, rote-learnt to no purpose, and in many cases just a repetition of primary experience. It was quite common to come across fractions taught each year in the same way, from the same book, to the same children, for the duration of their school life. Graphs were the high water mark of advanced mathematics. But the schools were not to blame if the staff were not available to cope with the problem. The enlightened few set about emancipating the curriculum until by 1956 there were some schools putting in their better pupils (about 15 to 20 from a school of 500) for G.C.E. and other similar external examinations. This led not only to be the raising of the standard of mathematics taught, but also meant that teachers began to plan their courses with the attitude that their children had a future after the age of 15. The success of these efforts led to the inclusion of more and more mathematics in the curricula of all forms and with freedom from external constraints some secondary modern schools are doing interesting work of high standard and quality, many including calculus in the third year. In the last two years the rate of development has increased enormously so that most schools have a G.C.E. form of about 25 pupils. It is not surprising that advisory literature has not yet caught up with the schools, for there have only been two previous books about the secondary modern mathematics course: one which has not caught up with 1945 (to be reviewed later) and one by Mr. James of Redland which copes with the B streams of three years ago.

We looked forward to this report with some expectation of learning about recent developments. In some directions expectations have been fulfilled, whilst in others there is a feeling of disappointment. The report has almost nothing to say about the "A" children.

The report falls into two parts: Purpose, Approach and Organisation in the first and Mathematical Content in the second.

The first part, 73 pages long deals, most comprehensively with methods of organisation covering remedial work, aims and scope, organisation and the use of practical topics. It rightly lays stress on the importance of something new when the children enter and illustrates how "Myself and my New School" can serve as a centre of interest to unite statistics, mensuration and arithmetical attainment. It is important to make preliminary diagnoses through the medium of a child's activities rather than through his response to a test of mechanical skill. Air Travel, the School Garden and Housecraft are treated in some detail to give the spirit of practical methods. The ensuing work on remedial matter emphasises pattern in number. Organisation concludes this section and a head of department will find good suggestions. There is nothing new in the proposals and one is struck by the need for more thought about the way to select topics and to relate them to mathematics and the other subjects of the school curriculum. One feels that a Syllabus formed by projecting mappings of mathematics on different planes, onto other planes might

serve better the muse of integrated mathematics than a sequence of topics which are on account of their sequence studied seriatim in time.

The second part of the book is one of the best treatments of stage A geometry and algebra which has been published. There are brief sections on mechanics and statistics but their brevity makes them mere mentions. Surveying, Navigation are given a full treatment. Having said that, one has to admit that this section is sadly lacking in mathematics and vision. The report might have given treatments of the ideas of calculus and coordinate geometry analogous to the infant treatment of number and mathematics which one could introduce and which would help the children to understand something of the underlying rationale of the various industries which would employ them. There is a lack of stage B collection of results into a pattern which has some reasonable basis however slight. There is no dovetailing of topics. One has the feeling that the report is concerned to reinforce existing textbooks rather than to break new ground. It is in fact rather behind what the schools themselves are developing. But, having said all this, it remains a land mark in the teaching of secondary modern mathematics. It will find an honoured place on staff-library shelves and will be well used by the teachers. It will serve as a valuable source for much of the B and C stream mathematics. Nevertheless, many teachers will bemoan its lack of forward vision, which reaches out to a new concept of secondary mathematics, stressing mathematical ideas and principles in a form which enables modern developments to be glimpsed if not appreciated, and at the very least will ask why the Jeffery report has had little influence on it. To quote the Spens Report of 1938 we must "treat mathematics as a science in which the topics are chosen so as to develop a grasp of mathematical ideas, and that these topics should be suggested and introduced by the examination of practical questions which in their day have been of urgent interest and utility to man in his affairs." It is perhaps in realising this suggested development that our own association might supplement the efforts of the Mathematical Association.

C.H.

(*Mathematics in Secondary Modern Schools* is published for the Mathematical Association by G. Bell & Sons, Ltd.)

CORRESPONDENCE

Sir,

Mr. Sillitto's article on pp. 25-27 of No. 10 of our Bulletin prompts me to make the following remarks:

1. Though questions on page 25 are of interest and must have occurred to anyone playing with the rods, the method used by the author does not seem to belong truly to the structures suggested by the rods. These are only whole numbers when measured by the white rod and because their cross-section is one square cm. we can pass from length to area. The true meaning of a cross as a product is that we use the length as an operator acting on an object that is, for whole number work, the white rod. If we only use a cross then one rod is an operator and the other an object. To have a product we must have two operators and replace the result by another rod whose length again operates on the white rod. This, in fact, is needed so that we

may prove that this notation for the product, as an autonomous operation, is both commutative and associative. So we have a tower as soon as we have a cross; this escaped Mr. Sillitto (see bottom of p. 26) and would have given his treatment a more adequate interpretation of what the rods do and cannot do.

2. In fact his whole article appears to use the rods for a purpose he has in his mind and not for what they permit. For instance when he says that y^3 is equal to 5 (p. 27), the only thing he can draw from it is $y = \sqrt[3]{5}$, which is a definition and not a numerical result. While if we see that we can form the powers of any number by using the towers made of one type of rod and write as exponent the height of the towers (again only possible because the rods have cross-section equal to one, and the rods are multiples of the white ones) we can relate each power to any other using fractional indices (as is mentioned at the end of the article, and as I showed years ago in several public meetings). My objection is that Mr. Sillitto's considerations are all in his mind already, and that he projects them into the rods instead of drawing them out of these. They become an aid to show what he wishes rather than a set of rods which is a model of a set of rational numbers with all the facilities of picking up the information they so easily hand out.

3. My objection to the treatment of fractions is still stronger, for Mr. Sillitto thinks that my treatment is inadequate and goes on to use a treatment that is half in the rods and half in his mind, since he always refers the 'product' of the numerator to a product he chooses for reasons clear to him. My position is that the product of fractions is to be defined as the result of a special composition of operators or of ordered pairs and that in order to maintain what is accepted for whole numbers (operators too, or ordered pairs with one as the measuring unit) we replace 'of' by 'multiplied by' and talk of multiplication (see my Book VI 'Arithmetic'). Since division is repeated subtraction or the inverse of multiplication we can introduce it in two ways, the second only after multiplication, the first only when fractions are things (wholes) and not operators. In both cases we have a difficulty to meet squarely rather than through the use of what the readers know.

Yours &c.,

C. GATTEGNO

TELEVISION IN SCHOOLS.

The School Broadcasting Council for Great Britain recently published a report on B.B.C. School Television Broadcasting during its first five terms. During this period there were two series of programmes on mathematics (both in the Autumn Term of 1958).

The period is clearly to be regarded as an experimental one, both from the point of view of the Television Service and of the Teacher. Here is a new medium: where lies its strength, where its weakness, and what of its future development? The basic principles underlying the policy planning of the first few years have been to meet the educational needs of the schools and to select suitable material for the medium. One important point which had to be remembered was the lack of receivers in schools and consequently programmes were aimed largely at the 11 to 15 years age-group to encourage L.E.A.s to concentrate their equipment in schools with this age range, thereby ensuring a reasonable sized audience for each programme.

It would appear that a number of primary schools are equipped with sets and have taken programmes successfully. The main body of viewers, however, comes from the secondary modern school; the proportion of Secondary Modern Schools to Grammar Schools equipped with sets is about five to one. But we suspect that this is not the only reason that the report soberly states that "Grammar Schools have shown rather less interest in television than have Secondary Moderns."

It is encouraging to learn, however, that where facilities existed, Heads of Schools have gone out of their way to give the service every chance in planning their time-tables and curricula, and that discerning teachers realised that they themselves had much to learn about using the programmes effectively; teachers work in preparing for, and following up broadcasts "varied from the non-existent to the very thorough, from the humdrum to the inspired."

The number of schools in the U.K. known to have viewed between September, 1958 and February, 1959 was 850; the largest number viewing any individual B.B.C. series was 569, the lowest 223. The series which produced the highest number of viewing schools watching it was a Natural History series; 66 per cent of all schools viewed. The series which produced the lowest was the second Mathematics series; 25 per cent of all schools viewed.

Considering the two mathematics series in detail, the report starts by saying, "This was obviously difficult territory *in which the possibilities of visual presentation otherwise than by blackboard and chalk were largely unexplored*" (our italics). After such a statement, one begins to suspect the reason why the mathematics series produced the lowest viewing score! The films of Nicolet and Fletcher, the models of Peskett and Swinden, and the many demonstration lessons and lectures given by a wide variety of speakers at A.T.A.M. Meetings all point a way for an effective use of this new medium. It would seem that the writers of the report (and possibly the producers of the programmes, too) are completely unaware that anything has been done to bring teaching aids to the help of the mathematics teacher.

The two mathematics series which were broadcast were *Shape around us*—an introduction to geometry through measurement, the triangle, the circle, loci, and similarity—and *Counting and drawing*—intended to illustrate the application of graphical work to statistics of various types. The report states that there was evidence frequently of television's capacity to illustrate a mathematical idea simply and vividly and in many classes there were signs of a renewed stimulus to work in mathematics, or even of a new attitude to the subject.

After reading the report it becomes clear that certain factors must be borne in mind in the future development of mathematics television presentation. Firstly it is necessary for the teacher to have very full details in advance of what it is hoped to do, and the teacher needs to spend time in careful planning and preparation of work before and after the broadcast to be able to exploit the medium. Secondly, it is essential to inspire pupil activity in the material presented; television so easily produces passivity on the part of its audience. Thirdly, a great problem which faces the planners of the programmes is the difficulty of matching pace and treatment (as well as content) to different pupil abilities and rates of comprehension. Finally, it is necessary for the planners to survey other mathematical teaching aids and make television programmes complementary to, and an extension of these.

"It appears . . . that the main function of television programmes in Mathematics is to provide a jumping-off ground for the teacher's own work. The programmes offered so far have certainly required the support of a teacher of at least average mathematical competence; and it would be quite premature to suggest that television can help make good any deficiency of specialists in this field. What it can do is to provide inspirational stimulus for both teachers and taught." In other words, the successful use of television requires *active* co-operation of broadcaster, teacher and pupil. C.B.

BOOK REVIEWS.

Dynamics.

A. E. Short. University of London Press. pp. 384. Price 30/-.

"Many students have felt the need for a Dynamics which makes full use of vector analysis, etc." This is the way in which the author begins his preface, and he has gone a long way and has done a fair job toward fulfilling the need. It is a book which does not shun the use of calculus and makes good use of vectors.

Part I starts with a short section on vectors and then gets on with the kinematics of a point, straight line motion and motion in a plane (hodograph and circular motion and derivative of a vector). Part II continues with the dynamics of a particle and introduces the (to me) novel concept of the Kineton, the vector quantity, $m\,f$. I do not think any revolutionary simplification ensues as a result and the term is used so infrequently as to throw doubt on its introduction. There follows the traditional treatment of work and energy (vector methods mean a precise definition of "work done" and the scalar nature of energy is thus exhibited), Impulse and momentum (including a nice treatment of billiard balls), projectiles and simple harmonic motion. There is however the inclusion of motion in a resisting medium, orbits and gravitation, and some examples of changing momentum (chain, raindrop and rocket) which adds another touch of novelty. Perhaps the traditional treatment of orbits might have been enlivened with some twentieth century problems such as Rutherford scattering, Sputniks, lunar probes, etc.

The third section deals with the motion of a rigid body and the parallel treatment by conventional and vector methods brings out to some degree the elegance of the latter. Vector methods really show their worth when there is simultaneous rotary motion in two directions in the general cases of rigid body motion. One feels that the treatment doesn't quite come off with the limitations imposed by the book. Perhaps the author's purpose might have been better served if he had not tried to produce the familiar variety of mechanics text-book and tried a more original arrangement and selection.

The book has the main fault of all mechanics text-books: it does not make the development from physical experience seem possible; it doesn't give any adequate discussion of the historical and physical background, which led to the development of the various concepts such as energy in its various forms, or momentum, and which enable a student to develop a fuller understanding by his own reading, unencumbered by the necessity of the interpretation of a third person. There is some

ground for believing that potential and motion might be more fundamental starting points than force and acceleration.

Nevertheless one must not under-estimate the magnitude of the step forward that this book emphasises, a step which those who remember Carey and Proudman's effort in 1925 will appreciate. C.H.

Fallacies in Mathematics.

E. A. Maxwell. Cambridge University Press, 95 pp. 13s. 6d.

Human folly can amuse and exasperate by turns, but here it is displayed so that it may instruct. The study of fallacies must be as old as the study of logical deduction, and one of the lost books of Euclid was devoted to "Pseudaria," but the part which it plays in most courses of systematic instruction is negligible. This is a pity, for how can a student learn to distinguish between the true and the false unless he sees the two displayed together?

We find here a rich variety of fallacies in geometry, algebra, trigonometry and calculus seasoned with diverting mistakes and pure howlers. The discussion of the fallacies goes surprisingly deep and often displays subtle points which are easily overlooked. The "isosceles triangle fallacy," for example, leads to a consideration of the deficiencies of Euclid's system of axioms.

Dr. Maxwell is the unchallenged master of fallacious argument (if he will pardon this way of putting it), and his book must not be regarded merely as a diversion, a Chamber of Horrors, a manual of oddities. University teachers often complain that pupils leave school equipped with techniques but with little idea of what mathematics is really about, and also that they show poor appreciation of the need for logical precision.

The discussion of fallacies might help, in a small way, to remedy these deficiencies, and by showing mathematics in sickness this book can help teachers to show it in health. T.J.F.

Recurrence Relations.

T. J. Fletcher.

Nomograms.

C. V. Gregg.

Mathematical Pie Ltd. Each 24 pp. 1s.

One of the difficulties in teaching Sixth Form pupils at the present time is encouraging them to undertake some wider reading around their subject. For the average pupil of the "television age," sitting down to read a book such as Clifford's *Commonsense of the Exact Sciences* or Courant and Robbins' *What is Mathematics* demands an effort of will that simply cannot be met. These pamphlets of a mere 24 pages length offer some hope of getting pupils to read something beyond the standard textbooks, and as such—if for no other reason—are to be welcomed.

Their presentation is straightforward and concise, ample reference is made to practical applications to create an interest and show the wide uses of the subject, and by either bibliography or by further questions to consider, the pupil is encouraged to follow up the brief material of the pamphlet by some wider reading.

Recurrence Relations gives an elementary survey of the solution and applications of recurrence series. Some of the applications are in the realms of science and computing; others are interesting questions of the puzzle type. The pamphlet may have benefited from some examples to work oneself; one has the feeling that examples from Mr. Fletcher would have been above the usual run of textbook examples. Also the subject could have been related more to topics with which the Sixth-former is already familiar; for example, most Sixth form courses now include some differential equations of the second order, and the relation of the auxiliary equation in series work to that in differential equations could be brought out.

Nomograms gives an introduction to the construction and use of these mathematical aids. The need for the various methods of scaling are considered, and the treatment is clear and well illustrated. Nomograms are given for many common relations such as simple and compound interest, trigonometrical formulae, Pythagoras' Theorem and mensuration of cylinders. A brief bibliography is appended.

To sum up, each of these booklets is well worth a "bob" of any Sixth former's money! C.B.

Pre-Senior National Certificate Drawing Course.

By S. H. Glenister. G. G. Harrap. pp. 160. 8s. 6d.

Engineering Drawing.

By S. H. Glenister. G. G. Harrap. pp. 109. 10s. 6d.

These two books are reviewed together because they are in a sense companion volumes but each provides an autonomous course. Perhaps the greatest appeal to the pupil is provided by the fact that most of the drawing exercises are "real life" and in many cases are illustrated by photographs of the components to be drawn in their actual position on an assembly, e.g. brackets on cutters (Pre-Senior) and engine suspension trunnion shown on an automobile engine assembly.

The pre-senior course contains a sensible progression of exercises, the usual geometry (regular polygons, similar figures, circles and ellipses), simple orthographic, oblique and isometric projections and developments of the simpler solids. It should prove useful for the growing number of secondary modern schools taking up geometrical drawing and should lead to many an interesting mathematical discussion in justification of the practices suggested. It would also be useful in the school library. Evening Institutes will also find it useful for their weaker classes although many would perhaps prefer to recommend a more meaty text book.

The Engineering Drawing, after a brief summary of geometry gets down to engineering. It covers the usual projections and deals with brackets and bearings, pictorial projection, projection of inclined surfaces, and screw threads. Each section has a brief explanatory passage about the use of what is being learnt and the drawings are accompanied by photographs of the real thing. The items are all in common use and appear to this reviewer to be likely to catch the eye and interest of the average schoolboy. It is eminently suitable for its purpose, S.1. and G.C.E. It can be recommended as a text and as a library book. C.H.

A.T.A.M. DIARY.

1959

Nov. 7th—Middlesex Branch. *Mathematics in Primary Schools.*
Details: Miss J. Blandino, 9 Barnhill Road, Wembley Park.

Nov. 28th—Cambridge Group. *Primary School Syllabus. Third Year Course.*
Details: Mr. N. Reed, Village College, Swavesey, Cambridge.

Nov. 27/28th—Basildon, Essex. *Mathematics in Primary Schools.*
Details: Conference Secretary.

Dec. 5th—Hitchin. (In conjunction with Assoc. of Maths. Teachers in Herts.; N. Herts. Branch).
Details: Mr. K. S. Maynard, Boys' Grammar School, Hitchin.

1960

March 12th—Colchester. *Mathematics in the Primary School.*
Details: Conference Secretary.

March 18/20th—Blackpool Conference and Annual General Meeting. (See below).

BLACKPOOL CONFERENCE 1960 AND ANNUAL GENERAL MEETING.

The 1960 Blackpool Conference will be held at Redman's Park House Hotel, North Promenade, from Friday, March 18th to Sunday, March 20th. This year the A.G.M. of the Association will be combined with the Conference. The usual team of speakers has been assembled and the theme will be

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Detailed programme is now available from Mr. C. Birtwistle, 1 Meredith Street, Nelson, Lancs. (Please enclose 2d. stamped addressed envelope). As demand for places usually exceeds accommodation available, early application is advised.

INCOME TAX: IMPORTANT ANNOUNCEMENT.

It is now permissible to claim relief of income-tax on the annual subscription to the Association, and members may claim this allowance commencing with the current financial year (i.e. April, 1959 to April, 1960).

In a letter to the Treasurer of the Association, H.M. Inspector of Taxes states that the Association has been approved under Section 16, Finance Act, 1958, "and that the whole of the annual subscription paid by a member who qualifies for relief under that section will be allowable as a deduction from his emoluments assessable to income-tax under Schedule E." Application for this allowance should be made to one's local tax office on form P358.

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A. E. SHORT, *Senior Lecturer in Mathematics Borough Road College, Isleworth, Middlesex.*

No previous knowledge of the subject is assumed and every effort has been made to introduce each topic in as simple a manner as possible and every stage in the development is supported by a set of easy, straightforward questions. The book will, therefore, be found suitable for all levels of sixth-form work in schools, while, at the same time, many who are studying for a degree will find it useful in the earlier part of their course.

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